A PROBABILISTIC TIME-REVERSAL SPATIAL CONVERGENCE METRIC: FDTD VALIDATION USING A MODIFIED MEEP IMPLEMENTATION

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Abstract

We present a novel time-reversal spatial convergence metric, inspired by probability theory and the electromagnetic wave potential energy density, and apply it to several FDTD time-reversal simulations based on a modified Meep implementation. The proposed metric is a convex-quadratic function of time, attaining its minimum at a time and a value corresponding to the source power expected value and standard deviation.

In the electromagnetic time-reversal cavity [1], a sensor spanning a full solid angle around a source allows to focus the EM fields at the original source location. This principle has been applied for both wave focusing and imaging. Regarding the focusing of the time-reversed waves, two issues remain: first, how to determine when the optimal (if any) focusing happens; second, how to meaningfully compare the "quality of focusing" of EM waves (i.e., the degree to which waves concentrated at a single location) across different scenarios. The latter element is central to super-resolution techniques.

State-of-the-art methods involve determining the local maxima or comparing the maximum to the side lobes [2, 3], or computing the entropy of the electric field [4]. The first two methods might not be appropriate for narrow-band signals. Also, the entropy lacks interpretability for EM waves while also suffering from an oscillating behavior, making optimal focusing hard to determine. In this paper, we propose a new spatial convergence metric based on probability theory and EM energy density and validate it in an FDTD simulation involving both wide- and narrow-band sources.

We base our metric on the EM energy density u_{EM}, which describes the density of potential energy carried by the EM fields. Its instantaneous value at time t and position $\mathbf{r}$ in a homogeneous, isotropic, lossless and passive medium of permittivity ε and permeability μ is given by

$$u_{\text{EM}}(t,\mathbf{r}) = \frac{1}{2\epsilon}|\mathbf{E}(t,\mathbf{r})|^2 + \frac{\mu}{2}|\mathbf{B}(t,\mathbf{r})|^2 \tag{1}$$

As long as the field potential energy is conserved, the integral

$$U_{\text{EM}} = \iiint_{\mathbb{R}^3} u_{\text{EM}}(t,\mathbf{r})\mathrm{d}^3\mathbf{r} \tag{2}$$

is independent of time. From this, we define the expected value of a function g with respect to the EM energy as

$$\mathbb{E}^t_{\text{EM}}[g] = \frac{1}{U_{\text{EM}}}\iiint_{\mathbb{R}^3} g(t,\mathbf{r})u_{\text{EM}}(t,\mathbf{r})\mathrm{d}^3\mathbf{r} \tag{3}$$

This amounts to considering $u_{\text{EM}}(t,\mathbf{r})/U_{\text{EM}}$ as a probability density function (PDF) in a three dimensional (spatial) probability space. We now define the expected x-coordinate of energy at time t as

$$x^t_0 = \mathbb{E}^t_{\text{EM}}[x] \tag{4}$$

and similarly for the y- and z-coordinates. In turn, the energy location standard deviation σ^t_x for the x-coordinate at time t is

$$(\sigma^t_x)^2 = \mathbb{E}^t_{\text{EM}}\left[(x - x^t_0)^2\right] \tag{5}$$

This corresponds to our definition of inverse of the "quality of focusing" of EM fields in the x-coordinate. We can define an aggregate metric combining all three axes by computing an equivalent volume given by the geometric mean

$$\sigma^t = (\sigma^t_x\sigma^t_y\sigma^t_z)^{1/3} \tag{6}$$

To validate the present metric, we run a set of 3D FDTD simulations using the Meep solver [5]. The simulation domain is a homogeneous, lossless, isotropic and passive medium, with an x-polarized dipole source at the origin. We test four electric dipole moments: a modulated Gaussian pulse, two windowed sines (of two and four periods), and an asymmetric signal consisting of two sequential windowed sines of different amplitudes. We also vary the normalized frequencies ranging from $\frac{1}{2}$ to 2 (all quantities in Meep are normalized such that the speed of light in vacuum is $c = 1$, and we choose the base unit of length to equal one wavelength).

To perform the time reversal, we modify the Meep source code to scale the magnetic fields H and B by a factor -1 efficiently, thus reversing the Poynting vector. This scaling corresponds to time reversal under two conditions: first, there is no active source and no charge accumulation. Second, the domain is large enough to accommodate the

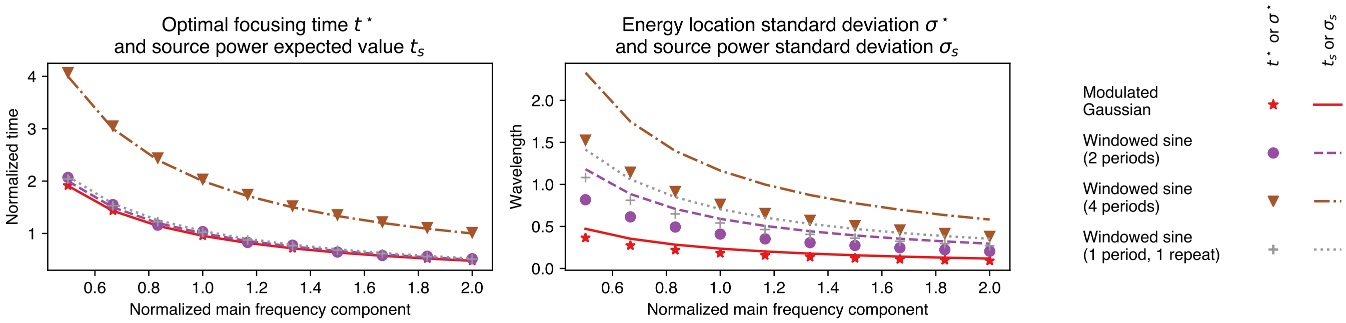

Figure 1. FDTD validation of the proposed spatial convergence metric for four electric dipole moments p as a function of the normalized main frequency component. The main frequency component corresponds to the frequency of the maximum of the spectrum. *Left*: optimal time of field energy focusing $t^\star$ given by the simulation and the corresponding expected time of source power t_s. *Center:* energy location standard deviation $\sigma^\star$ and source power standard deviation σ_s.

entire signal.

During the time reversal and at each time step, we compute the expected locations of energy x_0^t, y_0^t, z_0^t, then the aggregate energy location standard deviation σ^t from Equation (6). In all simulations, the shape of σ^t as a function of time is nearly quadratic, allowing to determine the optimal focusing time $t^\star$ and the optimal aggregate energy location standard deviation $\sigma^\star$ from the coordinates of the minimum of the quadratic fit.

Next, we define the expected time of source power as

$$t_s = \frac{\int_{-\infty}^{\infty} t p'(t)^2 \mathrm{d}t}{\int_{-\infty}^{\infty} p'(t)^2 \mathrm{d}t} \tag{7}$$

where p' is the derivative of the dipole moment. Likewise, the source power standard deviation σ_s is given by

$$\sigma_s^2 = \frac{\int_{-\infty}^{\infty} (t - t_s)^2 p'(t)^2 \mathrm{d}t}{\int_{-\infty}^{\infty} p'(t)^2 \mathrm{d}t} \tag{8}$$

The results show that the expected location of energy is always the origin, up to numerical errors. Also, the source power metrics t_s and σ_s relate closely to their field-based counterparts $t^\star$ and $\sigma^\star$. As seen in Figure 1, there is a very good agreement between $t^\star$ and t_s. If a source delivers a delayed signal, we can expect the same from the time-reversed field. Next, the energy location standard deviation $\sigma^\star$ and the source power standard deviation σ_s have the same inverse frequency dependence, and their ratio $\sigma^\star/\sigma_s$ lies in the interval $[0.6, 0.8]$ across all frequencies and dipole moments. Note that by reducing the FDTD cell size, this interval tightens. Thus, the proposed metric seems to be independent of the type of dipole moment and depends only on the source power standard deviation. As the main frequency component increases, the optimal focusing time and the energy location standard deviation decrease inversely. It is already established that time reversal works better with wide-band signals (i.e., those with low power standard deviation), which agrees with the results of the proposed metric.

To conclude, the proposed metric offers three advantages: the possibility to retrieve the original source location through the expected location of energy, an automated way to determine the optimal time of focusing $t^\star$ (e.g., allowing to reduce the duration of time-reversal simulations by stopping at $t^\star$), and a measure of the spatial spread of energy thanks to $\sigma^\star$.

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