STUDY OF THE CONVERGENCE OF THE CAUER LADDER NETWORK METHOD

W. Chen[1], T. Henneron[1], S. Clénet[1] and J. Zou[2]

[1]Univ. Lille, Arts et Metiers Institute of Technology, Centrale Lille, Junia, ULR 2697 - L2EP, F-59000 Lille, France
[2]Departement of Electrical Engineering, Tsinghua Universtity, China
wei.chen@ensam.eu

Keywords: Finite element method**,** Model order reduction, Cauer ladder network, Convergence

Abstract

Cauer Ladder Network (CLN) method is more and more used for order reduction of large numerical magnetoquasistatic model. It appears during the process of construction of the reduced model, loss of orthogonality of the vectors of the reduced bases which can lead to increase the error of reduction. To overcome this issue, a modified Gram-Schmidt process is introduced. The modified process of CLN construction is evaluated on a 3D magnetoquasistatic example in the frequency domain.

1 Introduction

Model order reduction methods are widely developed in computational electromagnetics because it can effectively reduce computational time while keeping accurate results on field distribution. Recently, the Cauer Ladder Network (CLN) method has been proposed by Kameari *et al.* [1] to reduce numerical model in magnetoquasistatics. This method enables to construct an equivalent electrical circuit as well as reduced bases where the reduced solution is sought. The construction of the reduced bases is based on an iterative process consisting in solving alternatively magnetostatic and current flow problems. In [2], it has been shown that CLN is equivalent to Padé approximation via the Lanczos process (PVL) for self-adjoint operators in linear space. In parallel, it has been shown that the Lanczos process leads to a loss orthogonality because of round off error in the finite precision arithmetic [3].

In this communication, we show that the original CLN method can lead to a loss of orthogonality of the vectors of the reduced bases. We propose an approach to keep the orthogonality by considering a modified Gram-Schmidt process and thus the convergence of CLN could be reinforced. The proposed method is evaluated on a 3D magnetoquasistatic example.

2 Cauer ladder Network

The Maxwell equations for a magnetoquasistatic problem in the frequency domain are:

$$\begin{aligned} curl\,\boldsymbol{H} &= \sigma\boldsymbol{E} \\ curl\,\boldsymbol{E} &= -j\omega\mu\boldsymbol{H} \end{aligned} \qquad (1)$$

Where $\boldsymbol{H}$**,** $\boldsymbol{E}$**,** μ and σ are the magnetic field, the electric field, the magnetic permeability and the electric conductivity respectively. We assume in the following that we have only one conductor Ω_c in a domain Ω supplied by a current *i*.

The CLN method aims at constructing two reduced orthogonal bases $(\boldsymbol{E}_{2n})_{n\in\mathbb{N}}$ and $(\boldsymbol{H}_{2n+1})_{n\in\mathbb{N}}$ satisfying:

$$\langle \boldsymbol{E}_{2n}, \sigma\boldsymbol{E}_{2m}\rangle_{\Omega_c} = (1/R_{2n})\delta_{nm} \qquad (2)$$

$$\langle \boldsymbol{H}_{2n+1}, \mu\boldsymbol{H}_{2m+1}\rangle_{\Omega} = L_{2n+1}\delta_{nm} \qquad (3)$$

Where δ_{nm} represents the Kronecker operator, R_{2n} and L_{2n+1} are the resistances and inductances of an equivalent electrical circuit presented in Fig 1.

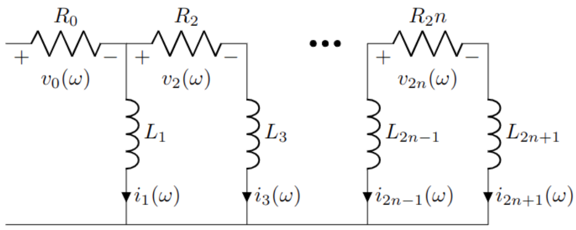

Fig 1. Cauer ladder circuit

The electric and magnetic fields are then approximated from the voltages $v_{2n}(\omega)$ and the currents $i_{2n+1}(\omega)$ defined in Fig 1. Such that:

$$\boldsymbol{E} = \sum_{n=0}^{\infty} v_{2n}(\omega)\boldsymbol{E}_{2n} \qquad (4)$$

$$\boldsymbol{H} = \sum_{n=0}^{\infty} i_{2n+1}(\omega)\boldsymbol{H}_{2n+1} \qquad (5)$$

The vectors $\boldsymbol{E}_{2n}$ and $\boldsymbol{H}_{2n+1}$ are calculated iteratively by solving alternatively magnetostatic and current flow problems [1]. The two static problems can be solved using a vector potential formulation (**A** or **T**) to keep the problem compatibility by imposing a divergence free source [4].

3 Application of the CLN method

We consider a conductor supply by a current *i* and surrounded by air presented in Fig 2. The permeability and conductivity of conductor are fixed to $4\pi \times 10^{-4}\,\mathrm{H/m}$ and $5 \times 10^{7}\,\mathrm{S/m}$. The mesh is made with 336 082 elements. The solutions of the equation systems derived from Finite Element (FE) **A** or **T** vector formulation, are the vectors of the reduced bases $(\boldsymbol{E}_{2n})_{n\in\mathbb{N}}$ and $(\boldsymbol{H}_{2n+1})_{n\in\mathbb{N}}$. To solve these equation systems, we have used either the direct solver MUMPS (MUltifrontal Massively Parallel sparse direct Solver) or an iterative solver Conjugate Gradient (CG). To quantify the orthogonality of the reduced basis ($\boldsymbol{E}_{2n}$), we calculate the angle $\varphi_{E,mn}$ given by:

$$\varphi_{E,mn} = \frac{\langle \boldsymbol{E}_{2n}, \sigma \boldsymbol{E}_{2m} \rangle_{\Omega_C}}{\sqrt{\langle \boldsymbol{E}_{2n}, \sigma \boldsymbol{E}_{2n} \rangle_{\Omega_C} * \langle \boldsymbol{E}_{2m}, \sigma \boldsymbol{E}_{2m} \rangle_{\Omega_C}}} = \frac{\langle \boldsymbol{E}_{2n}, \sigma \boldsymbol{E}_{2m} \rangle_{\Omega_C}}{\sqrt{R_{2n} R_{2m}}} \qquad (6)$$

We consider now a matrix **G** with its entries such that g_{mn}= 0 if $\varphi_{E,mn}$ <1% and g_{mn}=1 else. Theoretically, the matrix **G** should be diagonal since the reduced basis $(\boldsymbol{E}_{2n})_{n\in\mathbb{N}}$ is orthogonal (see (2)). In Fig 3, we can see that the reduced basis is not orthogonal and depends on the choice of the solver. As it was shown in [2], the CLN method is in fact equivalent to a Lanczos process which often leads to a loss of orthogonality [3]. In Fig 4, we present the evolution of the resistance and the inductance of the inductor obtained by the full FE model and the CLN method when using the MUMPS and GC solvers. It appears that the MUMPS solver enables to obtained good results unless the loss of orthogonality. However, we can see that the GC solver leads to incorrect results at high frequency and leads also to convergence issue.

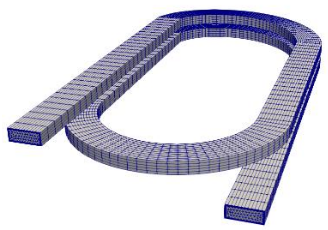

Fig 2. geometry of coil

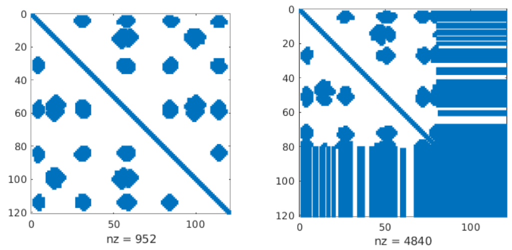

Fig 3. sparsity of **G** solved by MUMPS (left) and by GC (right)

3 CLN with modified Gram-Schmidt process

For the standard Lanczos process, one can use modified Gram-Schmidt process to force the vector $\boldsymbol{E}_{2n}$ to be orthogonal to all previous vectors $\boldsymbol{E}_{2i}$ with 0≤i≤n-1. Hence, we propose to use the modified Gram-Schmidt process. From the solution $\boldsymbol{E}'_{2n}$ of the vector potential formulation **T**, the new vector $\boldsymbol{E}_{2n}$ of the reduced basis is obtained by:

$$\boldsymbol{E}_{2n} = \boldsymbol{E}'_{2n} - \sum_{i=0}^{n-1} \frac{\boldsymbol{E}_{2i}}{\langle \boldsymbol{E}_{2i}, \sigma \boldsymbol{E}_{2i} \rangle_{\Omega_C}} \langle \boldsymbol{E}_{2i}, \sigma \boldsymbol{E}_{2n} \rangle_{\Omega_C} \qquad (8)$$

A similar approach is applied to the construction of the magnetic reduced basis $(\boldsymbol{H}_{2n+1})_{n\in\mathbb{N}}$. The CLN method converges even when using the GC solver, which gives now good results (see Fig.4).

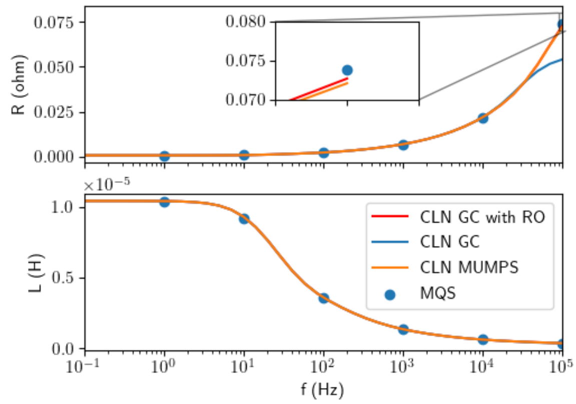

Fig 4. Evolution of the resistance R and the inductance L in function of the frequency given by the full FE model (MQS), the original CLN method with MUMPS (CLN MUMPS) and GC (CLN GC) solvers and modified CLN method (CLN with re-orthogonalization RO), both CLN composed of 120 stages.

5 Conclusion

The loss of orthogonality for the reduced basis built by the CLN method has been shown on a 3D magneto-harmonic example. It appears that it can be critical when using iterative solvers leading to the non-convergence of the CLN method. A modified Gram-Schmidt process has been proposed in order to circumvent this issue which has been successfully tested on a 3D example.

Acknowledgements

W. Chen was sponsored by the China Scholarship Council.

References

[1] A. Kameari, H. Ebrahimi. "Cauer ladder network representation of eddy-current fields for model order reduction using finite-element method", *IEEE Trans. Magn.*, vol. 54, no. 3, pp. 1-4, 2018.

[2] T. Matsuo, A. Kameari, K. Sugahara, and Y. Shindo, “Matrix Formulation of the Cauer Ladder Network Method for Efficient Eddy-Current Analysis,” *IEEE Trans. Magn.*, vol. 54, no. 11, pp. 1–5, 2018.

[3] B. N. Parlett and D. S. Scott, “The Lanczos algorithm with selective orthogonalization,” *Math. Comput.*, vol. 33, no. 145, pp. 217–238, 1979.

[4] N. Köster, O. König, and O. Bíró, “Proper Generalized Decomposition With Cauer Ladder Network Applied to Eddy Current Problems,” *IEEE Trans. Magn.*, vol. 57, no. 6, pp. 1–4, 2021.