INDUCTION HEATING ANALYSIS BASED ON CAUER LADDER NETWORK METHOD

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Abstract

The Lanczos algorithm is applied to derive the Cauer ladder network for induction heating analysis. The equivalent circuit obtained by the first Lanczos algorithm is a Cauer-I type circuit consisting of thermal inductance and resistance. Reapplying the Lanczos algorithm to the Cauer-I type circuit equation yields a Cauer-II type circuit consisting of a thermal capacitor and resistance, as in a typical thermal circuit. The temperature distribution can be reconstructed by superpositions of the basic functions corresponding to the circuit elements.

1 Introduction

With the technological development of power semiconductors and power electronics devices, high-frequency inverters have become inexpensive, and induction heating devices using high-frequency methods have become popular. As a result, many design sites are incorporating induction heating analysis in the design stage of induction heating devices. The Finite Element Method (FEM) is widely used as one of the leading methods for induction heating analysis because of its versatility and high accuracy. However, when the FEM is applied to high-frequency induction heating where the skin depth of the conductor becomes extremely small, the elements are divided according to the small skin depth, resulting in a vast matrix and, thus, enormous computational time. As a solution to this problem, Model Order Reduction (MOR), which approximates a system described by a large matrix as a small number of elements, is being actively studied by research institutes worldwide [1]. Among them, we have focused on the CLN method [2]. The CLN method is an approximate method that replaces the Maxwell equation for eddy current fields with a Cauer-type equivalent circuit to speed up electromagnetic field analysis. We propose that the CLN method could be applied not only to the Maxwell equation but also to the heat conduction equations. Furthermore, we expected that combining magnetic field analysis and heat conduction analysis using the CLN method would dramatically increase the speed of induction heating analysis.

This paper first describes the process of deriving a CLN for induction heating. Next, a CLN is derived for a model of heat generation density assuming induction heating, and the numerical examples are demonstrated to verify the method.

2 Formulations

The time constant of the magnetic field generated by an induction heating coil is much shorter than the time constant of heat conduction, and heat conduction does not have such a timely response. For this reason, in the following formulation, the CLN is derived using the time-averaged heat generation density $\overline{Q} = \frac{1}{2\sigma}\mathrm{Re}(\mathbf{J}\cdot\mathbf{J}^*)$ obtained by time harmonic eddy current analysis, as a source of the induction heating analysis.

2.1 Application of CLN to heat transfer analysis

Assuming that the heat generation density is Q, the temperature T, the thermal conductivity λ, the heat capacity per unit volume C, the Laplace operator $\triangle$, and the time derivative operator s, the heat conduction equation in continuous space is given by Equation (1).

$$-\frac{\lambda}{C}(\triangle T) + sT = \frac{Q}{C} \tag{1}$$

Defining $\mathbf{K}$ as $-\frac{\lambda}{C}\triangle$ in the FEM space and $\mathbf{N}$ as the identity matrix, the temperature distribution vectors $|\tilde{T}\rangle$, the heat generation density vectors $|\tilde{q}\rangle$, the heat conduction equation in the FEM space, Equation (1) can be written as Equation (2).

$$(\mathbf{K} + s\mathbf{N})|\tilde{T}\rangle = \frac{1}{C}|\tilde{q}\rangle \tag{2}$$

In reference [3], it has been shown that the CLN method can be applied to linear partial differential equations of diffusion type expressed using Hermite matrices $\mathbf{K}$, $\mathbf{N}$ as in Equation (2). Applying the Lanczos algorithm to Equation (2), $\mathbf{K}$ is approximated by a small-dimensional diagonalization matrix, $\mathbf{N}$ by a small triangularization matrix, and the circuit equations representing CLN in Figure 1 are generated. The CLN in which the resistor and inductor are repeatedly connected in series and parallel is called a Cauer-I type circuit. Theoretically, the Cauer-I type circuit continues infinitely, but for practical purposes, the circuit must be truncated at several stages.

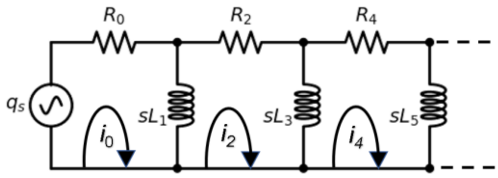


Figure 1. Cauer-I circuit for heat conduction

The CLN method generates the temperature basis functions $T_{2n+1}(r)$ corresponding to each circuit element simultaneously with the CLN generation. The temperature distribution $T(r)$ can be reconstructed by summing the product of i_{2n} and the corresponding basis functions $T_{2n+1}(r)$, as shown in Equation (3).

$$T(r) = \sum_{n=0} i_{2n} T_{2n+1}(r) \quad (3)$$

2.2 Conversion to the Cauer-II type thermal circuit

Reapplying the Lanczos algorithm to the Cauer-I circuit derived in the previous section, diagonalizing the $\mathbf{L}$ matrix and triangulating the $\mathbf{R}$ matrix, we obtain a CLN with resistors and inductors swapped with each other, as shown in Figure 3 (b). This CLN is called a Cauer-II type circuit. At this time, corresponding basis functions are generated automatically.

Furthermore, by performing 1/s conversion to the Cauer-II type circuit, it is possible to derive the equivalent circuit composed of resistances and capacitors such as Figure. 3 (c). This is a typical thermal circuit consisting of resistors in series and capacitors in parallel.

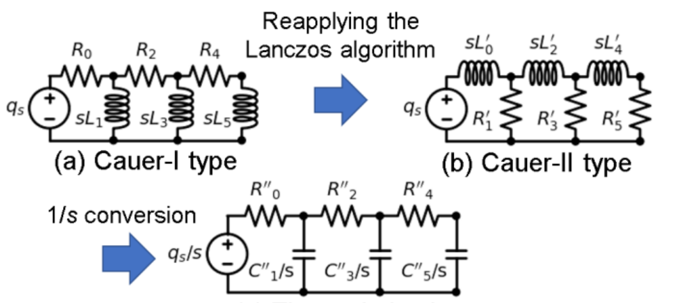


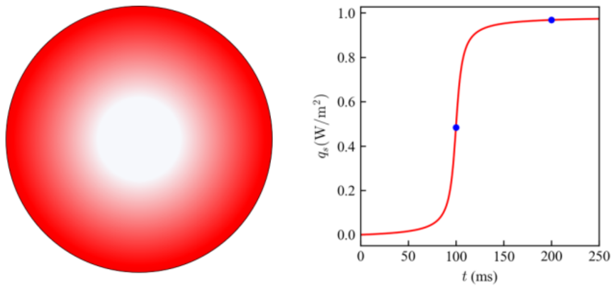
Figure 3. Derivation process from Cauer-I to thermal circuit

3 Numerical Result

Using a cylindrical conductor shown in Figure 4 (a) with thermal conductivity λ = 402 W/m K, heat capacity per unit volume C = 3.45 MJ/km^3, radius r = 3 mm, and heat generation density proportional to r^2 as an example, we analyzed temperature distribution using CLN and compared it with FEM. The time variation of the envelope of the heat generation density is also shown in Figure 4 (b). The initial temperature of the cylindrical conductor was set to 0 K, and the heat convection boundary condition was imposed on the edge of the cylindrical conductor.

$$\lambda \frac{\partial T}{\partial r} = -h(T - T_{ex}) \text{ on } \Gamma \quad (4)$$

where h is the heat convention coefficient, T_{ex} is the external temperature, Γ is boundary between the cylindrical conductor and the external domain. Here, T_{ex} = 0 K, h was 100 W/m^2 K.



(a) A cylindrical conductor given a heat generation density distribution $\propto r^2$

(b) Time variation of heat generation density

Figure 4. Time variation and spatial distribution of heat generation density

$T(r)$ obtained for a 2, 3, and 4-stage CLN is shown in Figure 5, along with FEM results for t =100 ms and t =200 ms, thus, the proposed method is verified.

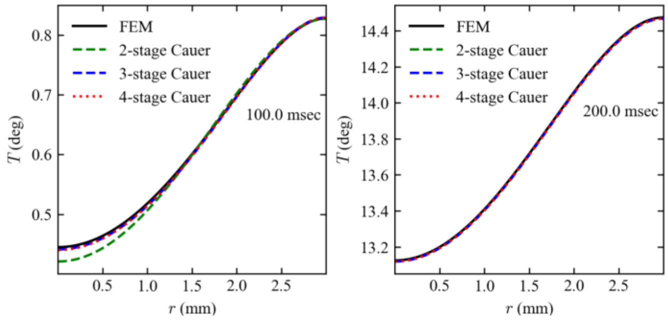


Figure 5. Comparison of the temperature distribution between CLN and FEM heat transfer analysis

Acknowledgements

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