SELECTION OF DATA POINTS FOR TRUNCATED SINGULAR VALUE DECOMPOSITION AS A METHOD FOR SHIM COIL DESIGN

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1 Abstract

We design shim coils to homogenize the magnetic field generated by realistic Helmholtz coils utilizing the Truncated Singular Value Decomposition (TSVD) method. The optimized shim coils shape depends on the selection of the evaluation surface. In this study, two sets of different evaluation points are investigated to design shim coils.

2 Introduction

The magnetic field generated by the coils in free space can be easily obtained using the Biot-Savart law. However, it is not straightforward to determine the shape of the shim coils from the target magnetic field. As one of those shape optimization techniques, we are focusing on the TSVD method [1] which uses a current vector potential $\boldsymbol{T}$ as unknowns. The current densities $\boldsymbol{J}$ are given by $\boldsymbol{J} = \mathrm{rot}\boldsymbol{T}$. We define magnetic field evaluation surfaces and current generating surfaces. $\boldsymbol{T}$ is restricted to having only normal components to the current generating surface. From the discretized $\boldsymbol{T}$, evaluation surfaces, and the Biot-Savart law, the response matrix $\boldsymbol{A}$ ($m \times n$ matrix) is constructed, where m is the number of discretized evaluation surfaces, n is the number of discretized $\boldsymbol{T}$, and $\boldsymbol{B}$ is the target magnetic field.

$$\boldsymbol{B} = \boldsymbol{AT} \tag{1}$$

When we apply the SVD method, $\boldsymbol{T}$ can be obtained as

$$\boldsymbol{T} = \boldsymbol{V}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^T\boldsymbol{B} \tag{2}$$

Equation (1) is generally singular and least square solution $\boldsymbol{T}$ is obtained by Equation (2) where V, Σ, U are singular value decomposed matrices. By truncating, or by taking only from the 0 -th to k-th eigenvalues, we obtain useful $\boldsymbol{T}$ for approximately regenerating $\boldsymbol{B}$. We define k as the truncation number of modes. It is known that the contour lines of the normal component of the $\boldsymbol{T}$ is equivalent to the coil windings [2].

In recent years, tabletop NMR systems have attracted much attention because they are inexpensive and easy to handle [3-4]. Our aim is to construct a tabletop NMR system using Helmholtz coils as a magnetic field source. We have evaluated the magnetic field inhomogeneity of the Helmholtz coils considering the actual windings. To correct the magnetic field to be homogeneous, shim coils are designed with the TSVD method. In this paper, we show the importance of the selection of magnetic field evaluation surfaces to design shim coils.

3 Numerical analysis of realistic Helmholtz coils

Figure 1 shows the numerical model of multilayer winding Helmholtz coils considering actual windings. The Helmholtz coils are modelled as a set of straight wire segments, and the numerical integration of the Biot-Savart law is employed. The length of each wire segment is about 8 mm. Because the axis of the pick-up coil is aligned perpendicular to the main magnetic field in the NMR application, the axis of the cylindrical sample space is also in the *x*-axis in general. The magnetic field generated by the Helmholtz coils is rather small. Therefore, the sample space is assumed to be larger than the conventional tabletop NMR to increase the signal intensity Magnetic field homogeneity in this work is defined by (3).

$$\text{homogeneity} = \frac{B_{z\max} - B_{z\min}}{B_{z\max}} \times 10^6 \text{ [ppm]} \tag{3}$$

The homogeneity of the magnetic field in the sample space is calculated to be 399 ppm, whereas the homogeneity of the ideal Helmholtz coils is 65 ppm.

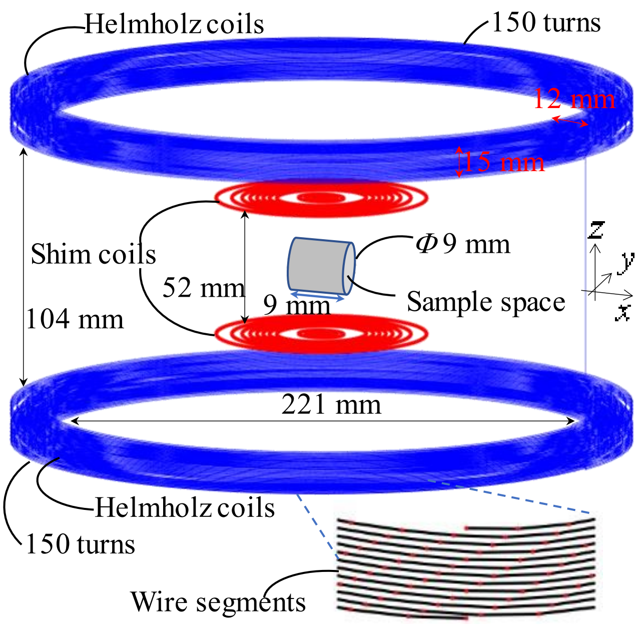

Figure 1 Coil configurations and a sample space

4 Design of shim coils by TSVD method

In the TSVD method, the evaluation points must be selected appropriately. Figure 2 shows the two types of magnetic field evaluation spaces. We have selected 5225 points in the sample space defined as type I, and 5055 points in the axisymmetric area covering the sample space defined as type II.

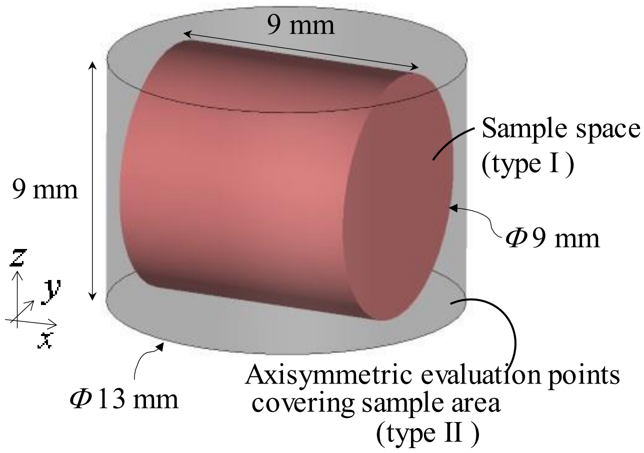

Figure2 Two types of magnetic field evaluation points

Figure 3 shows the dependence of the homogeneity on the truncation number of modes k with the evaluation points type I and II for the TSVD method. From the obtained contour plots of the current vector potential T, such as those in Figure 4, the shim coils can be designed. By adjusting the current flowing in the shim coils, the homogeneity in the sample space is evaluated. As known in the previous literature, the dependence of the homogeneity on k is the stepwise function.

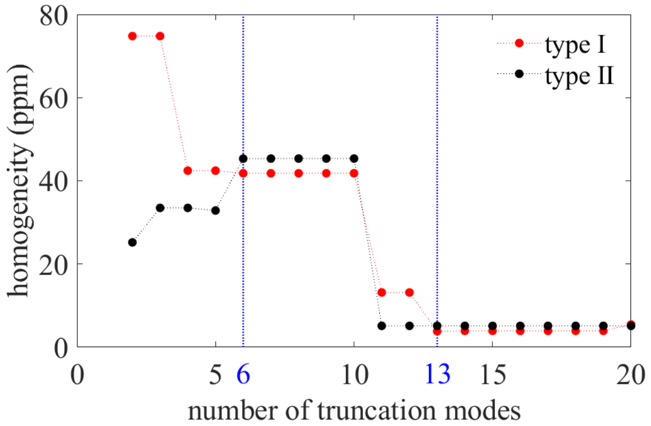

Figure3 Dependence of the homogeneity on k

Figure 4 shows the contour plots of T. Even though the homogeneity achieved with the same k is almost the same with the evaluation points type I and II, the contour plots are not symmetrical with type I, whereas symmetric with type II. Therefore, when designing practically useful shim coils, it is better to use Type II evaluation points than Type I because the shim coils obtained by Type II are easier to wind and still have almost the same homogeneity.

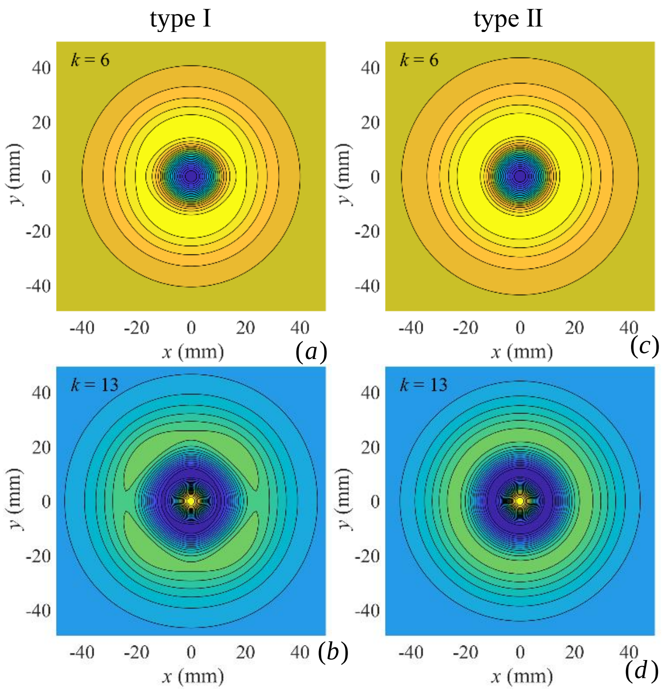

Figure 4 Contour plots of the current vector potential T obtained with type I (a), (b) and type II (c), (d) when $k = 6$ and $k = 13$.

5 Conclusion

The importance of the selection of magnetic field evaluation points is demonstrated when applying the TSVD method for designing a shim coil. The shape of the shim coils is significantly dependent on the evaluation points. In the presentation, experimental results will also be discussed.

Acknowledgments

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