TUNING OF THE TESLA CAVITY USING DERIVATIVE-BASED EIGENVALUE OPTIMIZATION AND TRACKING

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Abstract

In the design process of an accelerator cavity, optimization of the geometry parameters is necessary in order to tune the device. Finding the ideal set of parameters can be challenging and numerically expensive, as each evaluation requires the numerical solution of Maxwell's eigenvalue problem on the geometry at hand. The goal of this work is to efficiently employ a gradient-descent based optimization approach which takes multiple tuning objectives into account and ensures a correct matching of modes.

1 Introduction

For complex geometries such as the TESLA cavity, numerical methods are required to determine the structure's eigenfrequencies. As they sensitively depend on the geometry of the structure, geometry parameters need to be determined carefully in the design process of the cavity. While sampling approaches are straightforward to implement, they might need many evaluations of the model, to find a suitable set of parameters. In this work, we formulate an optimization problem which takes the requirements of cavity design into account.

2 Isogeometric Analysis

We discretize the computational domain by Isogeometric Analysis (IGA), which is based on using the same basis functions for both the geometry representation and analysis, namely B-splines and NURBS. If we employ the same basis functions as were used in the construction of the CAD geometry, no geometry modeling-related error is introduced. B-spline curves are defined from control points and the B-spline basis functions. Tensor products of the B-splines allow for a representation of surfaces and volumes. Details for this particular model are given in [4].

3 Problem formulation

Starting from Maxwell's equations and assuming a bounded, simply connected parametrized domain $\Omega_{\mathbf{p}}$ and perfect electric conducting (PEC) boundary conditions on

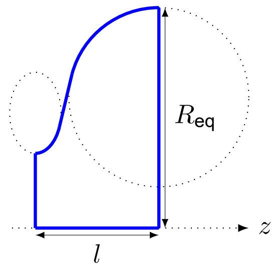

Figure 1: Design of the TESLA cavity.

$\partial\Omega_{\mathbf{p}}$, we derive the formulation

$$\begin{aligned} \operatorname{curl}(\operatorname{curl}\mathbf{E}) &= \lambda\mathbf{E} && \text{in } \Omega_{\mathbf{p}} \\ \mathbf{E}\times\mathbf{n} &= \mathbf{0} && \text{on } \partial\Omega_{\mathbf{p}} \end{aligned} \tag{1}$$

where $\mathbf{E}$ denotes the electric field strength, μ and ε the permeability and permittivity, respectively, and $\mathbf{n}$ the outwards pointing normal vector. By deriving the weak formulation and spatial descretization, we obtain a discrete generalized eigenvalue problem. We denote the solution by the eigenpair $(\lambda(\mathbf{p}), \mathbf{e}(\mathbf{p}))$ in dependence of $\mathbf{p}$ and the frequency $f(\mathbf{p}) = \sqrt{\lambda(\mathbf{p})}/(2\pi\sqrt{\mu\varepsilon})$.

As a first optimization goal, we would like to achieve a given eigenvalue λ_{ref} by determining the appropriate parameter $\mathbf{p}$. With the basic squared-error cost function

$$g(\lambda(\mathbf{p})) = (\lambda_{\text{ref}} - \lambda(\mathbf{p}))^2, \tag{2}$$

we can formulate the optimization problem

$$\min_{\mathbf{p}} g(\lambda(\mathbf{p})) \quad \text{s.t.} \quad \mathbf{K}(\mathbf{p})\mathbf{e}(\mathbf{p}) = \lambda(\mathbf{p})\mathbf{M}(\mathbf{p})\mathbf{e}(\mathbf{p}) \tag{3}$$

with the generalized eigenvalue problem given by the stiffness matrix $\mathbf{K}(\mathbf{p})$ and mass matrix $\mathbf{M}(\mathbf{p})$ as constraint.

4 Tuning of the TESLA cavity

We consider variations in three geometry parameters of the TESLA cavity [1] following the tuning procedure as described in [2], in order to tune the cavity towards the desired resonant frequency and field flatness of the accelerating mode. The tuned parameters are the length of the first half-cell l_1, the length of the last half-cell l_2 and the equator radius of the mid-cells R_{eq}, see Fig. 1.

Objective function The dynamics of the particle beam are affected by the electric field. Errors in phase and amplitude of the electric field cause beam degradation and losses [2] and the accelerating voltage should be maximized. Therefore, the tuning parameters need to be optimized in such a way that the accelerating electric field has the same magnitude in each cavity cell. Hence, we employ the field flatness criteria

$$\eta_1 = 1 - \frac{(\max_j |E_{\text{peak},j}| - \min_j |E_{\text{peak},j}|)}{\mathbb{E}(|E_{\text{peak},j})|}, \quad (4)$$

$$\eta_2 = 1 - \frac{\text{std}(E_{\text{peak},j})}{\mathbb{E}(|E_{\text{peak},j}|)} \quad (5)$$

introduced in [2] as a measure for an even distribution of the electric field peaks E_{peak} along the axis of the cells. To keep the field quality and as such the beam quality within acceptable limits, we require $\eta_1, \eta_2 \geqq 0.95$ for a well tuned cavity. To consider the field flatness in the optimization, we combine the former objective function (2) with the quality characteristics for field flatness, i.e.

$$g(\mathbf{p}) = (1-\eta_1)+(1-\eta_2)+\alpha(f_{\text{ref}} - f(\mathbf{p}))^2 + \beta\,||\mathbf{p}_{\text{diff}}||^2 \quad (6)$$

with the additional weight α. We penalize deviations $\mathbf{p}_{\text{diff}}$ from the original geometry and add a penalty term with $\beta > 0$ in order to find the local minimum which requires the smallest deviations from the nominal values.

Numerical results We use the Matlab function `fmincon` with the interior point algorithm to solve the optimization problem and compute the eigenvalue problem with the `eigs` function based on an Arnoldi method. When not providing `fmincon` with the gradients of the objective function, it approximates the derivatives by forward finite differences. In the full paper we show how shape derivatives with respect to the control points can be used as an efficient closed-form alternative.

We consider the TESLA cavity from three cells and with attached beampipes. To limit deformations, we permit variations of up to ± 2 mm for each parameter. The reference frequency was chosen as 1.3 GHz, which is the desired accelerating frequency for the TESLA cavity. For the objective function, we choose weight $\alpha = 10^{-15}\,\text{Hz}^{-2}$ and the penalty parameter $\beta = \frac{2}{3} \cdot 10^{-2}$. The thereby obtained optimal tuning parameters are shown in Tab. 1. We yield a field flatness with $\eta_1 = 0.9973$ and $\eta_2 = 0.9817$.

Parameter	nominal value [mm]	tuning [mm]
l_1	56.0	$+0.5573$
l_2	57.0	$+0.6709$
R_{eq}	103.3	$+0.5880$

Table 1: Optimal values for the tuning parameters.

5 Eigenvalue Crossings

During the shape optimization of a cavity its eigenvalues may cross. This is illustrated in the following for a pillbox cavity, see Fig. 2. When selecting the first eigenmode at $r = 5.5$ cm, which corresponds to the TM010 mode, and optimizing towards a target frequency of 2.5 GHz, we would optimize a different mode if further considering the first one, namely the TE111 mode, when traversing the crossing of the eigenfrequencies at $r = 4.92$ cm. This issue is also described e.g. in [3]. This can be mitigated by checking the correct matching in the optimization iterations. The approach is based on [4] and details are given in the full paper.

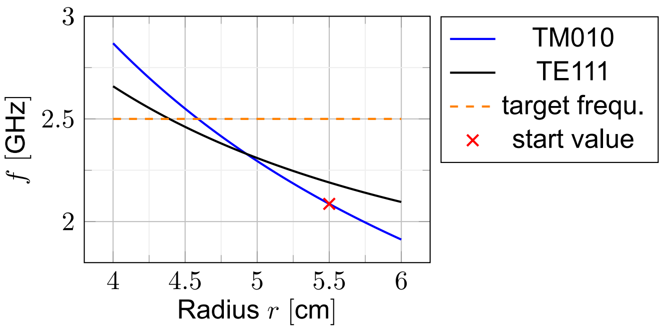

Figure 2: Eigenvalue crossing in the cylindrical pillbox cavity when modifying the radius.

6 Conclusion

In this paper we have presented an efficient way to tune the TESLA cavity towards a resonant frequency and field flatness while keeping the deviations small. This approach can easily be adapted to also be used in the design of new cavities, taking into account further design goals. The optimization is enhanced by using shape derivatives and eigenvalue tracking.

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