A DISCRETE VARIABLE BASED METHODOLOGY FOR TOPOLOGY OPTIMIZATION CONSIDERING MANUFACTURING ERRORS

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Abstract

Manufacturing errors (ME) are ubiquitous and inevitable in product engineering manufacturing. However, the existing methods are hardly oriented to the discrete-variable-based modelling methodology for the topology optimization (TO). In this regard, a novel methodology based on morphological operations and random filed (MORF) is proposed for the discrete-variable-based topology optimization procedures to consider MEs. Morphological operations are firstly introduced to generate the geometrical variation. Moreover, the dimension of the structuring element in the morphological operations is set as the output of the random field function. Using the proposed approach, MORF is capable of quantifying spatially nonuniform MEs rigorously. The numerical result has validated the proposed method.

1 Introduction

Topology optimization(TO) is to find the optimal materials distribution of a device under some performance criteria in the initial stage of product manufacturing. For a real world device, the deviation of the manufactured topology from the computationally optimized one resulted from a manufacture tolerance will dramatically degrade the performance of the optimized design. Consequently, Robust Optimization (RO) considering Manufacturing Errors (ME) has become a hot topic in TO studies

One of the key issues in ROME is the appropriate representation of a shap or topological variation. As is well known, two different modelling methodologies, the discrete-variable-based and the continuous-variable-based ones, are commonly used in TO studies. Nevertheless, the existing techniques for ROME in TO are merely oriented to the continuous-variable-based modelling methodology, such as the SIMP and the LSM, while negligible efforts have been paid to the ROME in the discrete-variable-based TO. A continuous-variable-based TO method basically represents the geometrical deformation by disturbing the intermediate variables, and is obviously inapplicable to a discrete-variable-based modelling one [1]-[2].

In this regard, a methodology based on morphological operations and random fields (MORF) is firstly proposed to solve a discretely modelled TO problem considering MEs. Specifically, the geometrical deformation is regarded as the fluctuation of the interface between two materials, and is represented using the morphological operations conducted on the computationally optimized topology. The dimension of the morphological operator is defined as the output of a random field, enabling MORF to represent spatially nonuniform MEs. The numerical results have validated the ability of the proposed method to produce optimized topology to withstand MEs.

2 ME Representation based on Morphological Operations and Random Field

Dilation and erosion are two basic morphological operations. Dilation adds pixels to the boundaries of objects in an image, while erosion removes pixels on object boundaries. The number of pixels added or removed from the objects in an image depends on the size and shape of the structuring element (SE) used to process the image. Fig. 1 depicts intuitively the results of erosion and dilation operations on an image under a diamond-shaped SE. Obviously, the dimensions and shape of the SE jointly determine the pattern of the new image.

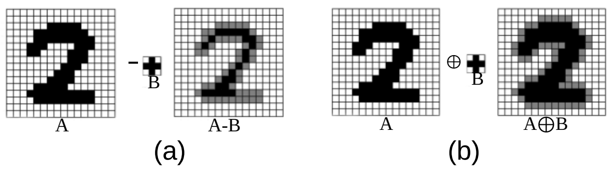


Fig. 1. (a) Erosion operation, (b) Dilation operation

In a discrete-variable-based TO, the ME could be interpreted as reordering 0/1 patterns in the neighbourhood of the boundary. In other words, some parts of the material tend to grow again, while some parts of the material tend to contract. Therefore, it is reasonable to represent the growth of the material by a dilation operation, and the contraction by an erosion operation. In practical engineering, the ME usually varies in space. To this end, the random field theorem is employed to generate a spatially nonuniform ME.

A random feld $h(\boldsymbol{x}, \theta)$, with θ belonging to space of random event Ω, is expressed using KL expansion as:

$$h(\boldsymbol{x},\theta)=\bar{h}(\boldsymbol{x})+\sum_{n=0}^{m}\delta\xi_n(\theta)\sqrt{\lambda_n}f_n(\boldsymbol{x}) \quad (1)$$

where $\bar{h}(\boldsymbol{x})$ and δ are the mean value and variance of $h(\boldsymbol{x}, \theta)$, respectively, m is the truncation order, $\xi_n(\theta)$ is the uncorrelated random variable, λ_n and f_n are eigenvalues and eigenvectors of the covariance function $\boldsymbol{C}(\boldsymbol{x}_1, \boldsymbol{x}_2)$ following:

$$\int_D C(\boldsymbol{x}_1, \boldsymbol{x}_2)f_n(x_2)\mathrm{d}\boldsymbol{x}_2=\lambda_n f_n(\boldsymbol{x}_1) \quad (2)$$

In this study, a squared exponential covariance function is used:

$$C(\boldsymbol{x}_1, \boldsymbol{x}_2)=e^{-|x_1-x_2|/L} \quad (3)$$

where L is the correlation length. Substitute (3) into (2), one can obatin

$$\int_{-a}^{+a} e^{-|x_1-x_2|/L} f_n(x_2)\mathrm{d}\boldsymbol{x}_2=\lambda_n f_n(\boldsymbol{x}_1) \quad (4)$$

After obtaining λ_n and f_n, the random field could therefore be approximated using Eq. (4).

To employ the random field $h(\boldsymbol{x}, \theta)$ to represent the nonuniform deformation in the neighborhood of the interface Bn, the mean value of $\omega(\boldsymbol{x}, \theta)$ is set to be zero, indicating that the deformations fluctuate around the original interface. The integral interval in Eq. (4) is discretized into K segments, which is equal to the number of the elements in Bn. In each realization of $h(\boldsymbol{x}, \theta)$, a vector containing K random variables will be generated. As showed in Fig. 2, for the ith element on the interface, a morpholigical operation with an SE (suppose a square shaped one) of the length equal to $h(i)$ will be operated. Specifically, a positive diameter infers a dilation, while a negative diameter infers an erosion. Fig. 3(a) and (b) show five realizations of $h(\boldsymbol{x}, \theta)$ and the corresponding deformations (black) on a 400×400 square interface (red), respectively.

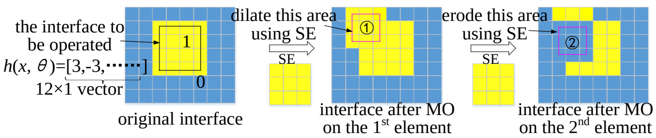
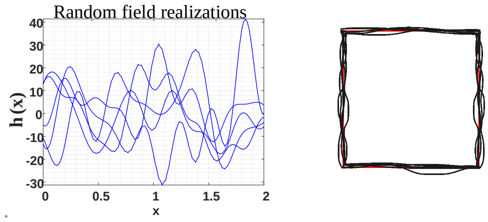


Fig. 2. Illustration of dilation and erosion based on random field



(a) (b)

Fig. 3. realizations of random filed and the corresponding deformed interface

3 Numerical results

To testify the proposed methodology, a magnetic actuator, as showed in Fig.4 (reference model), including a yoke, a coil, and an armature [10] is topologically optimized to maximize the magnetic force in a specific direction. The number of finite elements in the design region is 30×9=270. The input current of 1 A is applied to the coils with 400 turns.

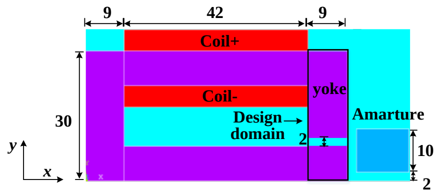


Fig. 4. Schematic diagram of the actuator

Fig. 5(a) shows the optimized results of the deterministic TO. Fig. 9(b)-(c) show the optimized results of the robust TO under θ = (L=0.5, δ=1) and θ = (L=0.5, δ=1.5), respectively (black area indicates the air). The magnetic forces are listed in Tab 1. Compared to the robust optimized topology, the air area (black) at the left lower corner of the design domain in the deterministic optimized topology is quite thin and tends to shrink caused by a geometry uncertainty (Fig. 5(c)). In this way, the magnetic flux will be blocked from reaching the armature, resulting in a dramatic drop of the magnetic force. While in the robust optimized topology, this area grows larger, thus insensitive to the geometry uncertainty.

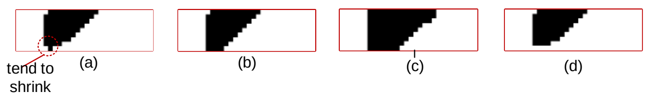


Fig. 5. (a) Deterministic optimized topology; (b) Robust optimized topology with L=0.5, δ=1; (c) Robust optimized topology with L=0.5, δ=1.5; (c) Distorted topology from (a)

Table 1
The Optimized Results

	Magnetic force (N/m)	Expected value(N/m)	Standard deviation
Deterministic result	-55.62(30.35%↑)	-28	668
Robust result	-52.82(23.79%↑)	-51.97	4.21

References

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