EXTENDED APPLICATION OF THE KELVIN TRANSFORMATION IN HIGH-FREQUENCY ELECTROMAGNETIC FIELD OPEN BOUNDARY PROBLEMS

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Keywords: finite element analysis, high frequency electromagnetic, open boundary problems, conformal transformation, Kelvin transformation.

Abstract

This paper discusses the extension of the Kelvin transformation to high-frequency electromagnetic problems. The Kelvin transformation is a coordinate transformation that maps infinite space to finite space, which can be regarded as a conformal transformation of Maxwell's equations. Using the concept of differential geometry, the metric and the spatial dependence of the material constant in the exterior domain can be obtained. When dealing with open boundary problems in radiated electromagnetic fields, the analysis domain is cut off by Perfect Matched Layer (PML). The PML usually needs to be located at least one wavelength away from the source. If the size of the radiating object is smaller than the wavelength, the total amount of the analysis domain becomes enormous. Therefore, by applying the concept of the Kelvin transformation and placing the PML within the exterior domain, numerical analysis can be performed with a realistic domain size.

1 Introduction

Bounded-domain solution methods for electromagnetic field analysis, such as the finite element method and the finite difference time domain method, cannot directly handle open boundary problems. Various methods have been proposed to emulate them [1]-[2]. Each method has certain advantages and limitations. These are called Asymptotic Boundary Conditions for low-frequency problems in which the effects of displacement currents are neglected and Absorbing Boundary Conditions for high-frequency electromagnetic problems. In this paper, the Kelvin transformation [3]-[6], which functions as an exact open boundary condition, is focused on among all the open boundary techniques. The analysis domain is terminated by a circle for a two-dimensional problem or a sphere for a three-dimensional problem, and the exterior domain is Kelvin transformed and connected to the analysis domain with unknown equivalent boundary conditions. In [6], the Kelvin transformation is reformulated to derive the conductivity, permittivity, and permeability in the exterior domain, conserving the conformal symmetry of Maxwell's equations. Utilizing the derived material properties, the materials can be both in the interior and exterior domains or even across the truncated boundaries.

In this paper, the idea of reference [6] has been extended to high-frequency radiation problems. For example, consider the problem of an antenna containing multipath from mountains and buildings, as shown in Figure 1. If the analysis domain is set up to include the ground, buildings, and mountains, the size of the analysis domain is vast, and an enormous computational cost is required. If the proposed method is applied to such a problem, the analysis domain can be limited to the region encompassing the antenna, and other scatterers can be included in the external region.

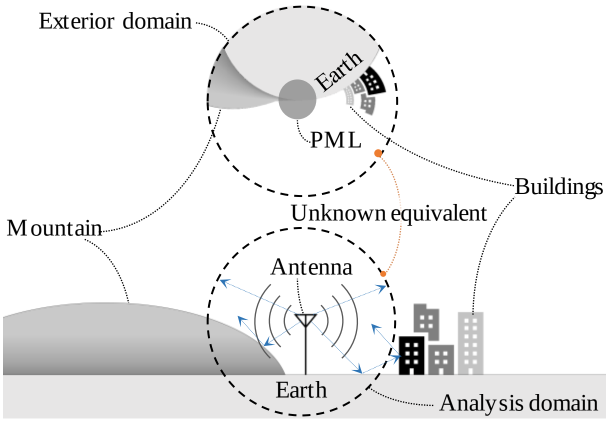

Figure 1: Concept of proposed Kelvin transformation

2 2D Formulation

The Maxwell equations in two dimensions are formulated using the z-component of the vector potentials A_z

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right)+\varepsilon\omega^2 A_z=0 \qquad (1)$$

where ε is complex permittivity, μ is complex peaceability, and ω is an angular frequency.

The spatial coordinates of the Kelvin transformation in two dimensions are given as

$$x'=(\frac{a^2}{x^2+y^2})x,\, y'=(\frac{a^2}{x^2+y^2})y,\, z'=z\,. \qquad (2)$$

The material properties (ε,μ) in the exterior domain is given by Equation (3) as in [6].

$$\begin{cases} \varepsilon'(x',y') = \left(\dfrac{a}{\sqrt{x'^2+y'^2}}\right)^4 \varepsilon(x,y) \\ \mu'(x',y') = \mu(x,y) \end{cases} \tag{3}$$

The simple Maxwellian PML is placed at the center of the exterior domain. The material properties of the Maxwellian PML can also be transformed using Equation (3), and the true center of the exterior domain is excluded to avoid the singularities.

3 Numerical verification of 2D example

Contour plots of the obtained vector potentials of the two-dimensional example are shown in Figure 2. A circular dipole source is placed above the perfect electric conductor ground, which has analytic solutions with Hankel functions of the second kind. The wave number $k = 2.0\,\mathrm{m}^{-1}$ is comparably small, but the PML is placed far enough away from the source in the exterior domain.

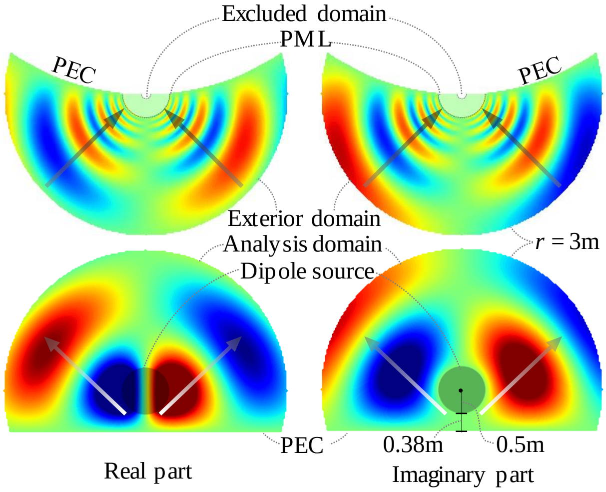

Figure 2: Contour plots of the vector potentials

Comparisons of the vector potentials along the x-axis obtained by the proposed method with the analytic solutions are shown in Figure 3. An almost perfect match has been observed, which indicates the validity of the proposed method.

4 Conclusion

The idea of the Kelvin transformation has been extended to high-frequency radiation problems. The proposed method is effective in the following two cases. 1) surrounding scattering objects are present; 2) radiating object is small compared to the wavelength and difficult to place the PML far enough. The numerical verification of the proposed method is performed with the two-dimensional example; however, the same discussion applies to the three-dimensional problems.

5 Acknowledgements

This work was supported by the JSPS KAKENHI Grant-Numbers JP20K04454.

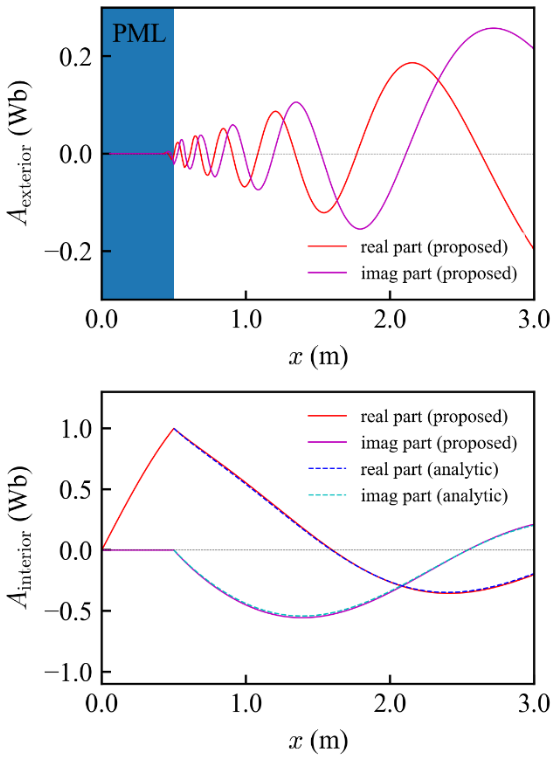

Figure 3: Vector potentials along the x-axis obtained by the proposed method and the analytical formula. An almost perfect match has been observed, which indicates the validity of the proposed method.

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