DATA-DRIVEN DETERMINATION OF B(H) CURVES OF IRON YOKES IN NORMAL CONDUCTING ACCETERATOR MAGNETS

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Abstract

The B(H) curve of yoke laminations may be an uncertain parameter in numerical field models of accelerators due to manufacturing variances or because sample magnetometer measurements may not be available. In this work, a method is derived to compute the B(H) curve of iron yokes, for a given set of material measurements on ring specimens and field measurements of the built magnet. Instead of using a closed-form expression for the B(H) curve, material measurements are used to derive a data-driven model that reflects the observed variances due to chemical compositions, heat treatment and cold working. To this end we make use of a truncated Karhunen-Loeve expansion. The parameters of the data-driven model are subsequently updated by fitting the simulated to the measured magnetic flux density in the magnet. It is shown that the proposed method can retrieve a previously selected ground truth B(H) curve that was used to generate the field data for the fitting.

1 Introduction

Numerical field computation is an established tool for the design of accelerator magnets and for the tracing of manufacturing errors in the built magnets. The numerical models depend on input parameters, such as the B(H) curve of the yoke material. Although the B(H) curve of toroidal material specimen can be measured [1], the exact magnetization curve of the magnet yoke might differ due to variations in the manufacturing process, stress levels or ageing processes. The determination of the B(H) dependence of a particular piece of magnetic material inside electric devices has been treated in [2] and of the yoke of normal conducting accelerator magnets in [3]. These approaches describe the B(H) curve by parametrized closed-form expressions and determine the parameters by solving an optimization problem that fits a simulated to a measured quantity.

Experience has shown that fitting these closed form expressions to the measured B(H) curves results in large residuals.

In this paper the closed form expression is substituted by the random field-based H(B) curve description derived in [4] from measured data using the truncated Karhunen-Loeve expansion. The resulting H(B) curve parametrization provides the (provable) best description according to the underlying manufacturing-related variations and their probability. Subsequently it is shown, for a test case, that the parameters can be determined by solving an optimization problem that fits the (measured) flux density of the corresponding magnet. In this way, the numerical model is tailored to a specific magnet as built and allows interpolative field prediction for excitation currents that have not been measured.

2 Magnetostatic Problem

The numerical model of the normal-conducting, iron dominated magnet on a domain D is based on the magnetostatic problem given by

$$\begin{aligned} \operatorname{curl}\mathbf{H} &= \mathbf{J} \quad \text{in } D \\ \operatorname{div}\mathbf{B} &= 0 \quad \text{in } D \\ \mathbf{B}\cdot\mathbf{n} &= 0 \quad \text{on } \partial D. \end{aligned} \tag{1}$$

Thereby, $\mathbf{B}$ is the magnetic flux density, $\mathbf{H}$ denotes the magnetic field strength, $\mathbf{J}$ is the electric current density with $\operatorname{div}\mathbf{J} = 0$ and $\mathbf{n}$ is the outward pointing unit normal. If the domain D is contractible, the magnetic vector potential $\mathbf{A}$ can be introduced such that $\operatorname{curl}\mathbf{A} = \mathbf{B}$. By incorporating the vector potential and the constitutive equation $\nu(\|\mathbf{B}\|)\cdot\mathbf{B} = \mathbf{H}$, the curl-curl formulation of the magnetostatic problem

$$\begin{aligned} \operatorname{curl}\nu(\|\operatorname{curl}\mathbf{A}\|)\operatorname{curl}\mathbf{A} &= \mathbf{J} \quad \text{in } D \\ \mathbf{A}\times\mathbf{n} &= 0 \quad \text{on } \partial D \end{aligned} \tag{2}$$

is obtained. The corresponding weak formulation is: Find $\mathbf{A} \in \mathcal{V}$, such that

$$\int_D \operatorname{curl}\nu(\|\operatorname{curl}\mathbf{A}\|)\operatorname{curl}\mathbf{A}\cdot\operatorname{curl}\mathbf{v}\ \mathrm{d}V = \int_D \mathbf{J}\cdot\mathbf{v}\,\mathrm{d}V \tag{3}$$

for all $\mathbf{v} \in \mathcal{V}$, with

$$\mathcal{V} := \{v \in \mathcal{H}_0(\mathrm{curl}; D) \,|\, \langle\mathbf{v}, \mathrm{grad}w\rangle_{\boldsymbol{D}} = 0\ \forall w \in H_0^1(D)\}. \tag{4}$$

3 Material model

For non-linear, isotropic and anhysteretic material the reluctivity function $\nu: \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is defined for $\mathbf{B} \neq \mathbf{0}$ by

$$\nu(\|\mathbf{B}\|) = \frac{f_{\mathrm{HB}}(\|\mathbf{B}\|)}{\|\mathbf{B}\|}. \tag{5}$$

Thereby, the material curve $f_{\mathrm{HB}}: \mathbb{R}_0^+ \to \mathbb{R}_0^+, B \mapsto H$ maps the intensity $B := \|\mathbf{B}\|$ of the magnetic flux density to the intensity $H := \|\mathbf{H}\|$ of the corresponding magnetic field strength. The requirements of f_{HB} and ν, to yield a unique solution of the weak curl-curl problem are found in [4]. Among others, the monotonicity of f_{HB} and the existence of a constant $\alpha > 0$ that serves as a lower bound of the derivative of f_{HB} is crucial.

Given a set of B-H data measurements

$$\{(B_1^1, H_1^1), \dots, (B_L^1, H_L^1), \dots, (B_1^K, H_1^K), \dots, (B_L^K, H_L^K)\} \quad (6)$$

of K specimens in L discrete points in the interval of B. For each specimen the discrete measurements are interpolated by a monotone cubic spline curve

$$f_k\colon I \to \mathbb{R}_0^+, \quad I \coloneqq [0, \min_{k \le K} B_L^k]. \quad (7)$$

According to [4], these curves can be seen as realisations of the random field

$$f_{\mathrm{HB}}\colon \Omega \times I \to \mathbb{R}_0^+ \quad (8)$$

also called f_{HB} by abuse of notation. Following [4], the random field is discretized by the truncated Karhunen-Loeve expansion

$$f_{\mathrm{HB}}^M(\omega, s) = \mathbb{E}[f_{\mathrm{HB}}(s)] + \sum_{m=1}^M \sqrt{\lambda_m} Y_m(\omega) b_m(s) \quad (9)$$

where (λ_m, b_m) are eigenpairs of the operator

$$T_{f_{\mathrm{HB}}}(u)(s) \coloneqq \int_I \mathrm{Cov}\big(f_{\mathrm{HB}}(s), f_{\mathrm{HB}}(t)\big)\, u(t)\, \mathrm{d}t \quad (10)$$

and the random variables Y_m are for $\lambda_m > 0$ given by

$$Y_m(\omega) = \frac{1}{\sqrt{\lambda_m}} \int_I (f_{\mathrm{HB}}(\omega, s) - \mathbb{E}[f_{\mathrm{HB}}(s)])\, b_m(s) \mathrm{d}t. \quad (11)$$

The eigenvalue problem is numerically solved by the Galerkin method and an approximation with radial basis functions. Furthermore, the expected value $\mathbb{E}[f_{\mathrm{HB}}(s)]$ is approximated by the sample mean $\hat{f}_{\mathrm{HB}}(s)$ of f_k. Thus, a parametrized H(B) curve $f_{\mathrm{HB}}\colon \mathbb{R}^M \times I \to \mathbb{R}_0^+$, that reflects the observed variations of the realizations f_k is given by

$$f_{\mathrm{HB}}(\mathbf{y}, s) := \hat{f}_{\mathrm{HB}}(s) + \sum_{m=1}^M \sqrt{\lambda_m} \mathbf{y}_m b_m(s). \quad (12)$$

Feasible upper and lower bounds $\mathbf{y}^{\min}$ and $\mathbf{y}^{\max}$ are estimated by inserting the realizations f_k in Eq. (11). In order to ensure the required monotonicity of the resulting H(B) curve with the intermediate value theorem the monotonicity is checked for each combination of upper and lower bounds, and they are adjusted accordingly if the monotonicity fails.

4 Determination of model parameters

The parameters of the parametrized H(B) curve $f_{\mathrm{HB}}(\mathbf{y}, \cdot)$ are determined by solving the following optimization problem (minimizing the deviation of the measured to the simulated vertical field component in the magnet at different positions p and current levels j):

$$\min_{\mathbf{y}_m \in [\mathbf{y}_m^{\min}, \mathbf{y}_m^{\max}]} \sum_{p \in P, j \in J} \left\| \mathbf{B}_y^{\mathrm{meas}}(p, j) - \mathbf{B}_y^{\mathrm{sim}}(p, j) \right\|^2$$

s.t. $\quad \mathbf{B}^{\mathrm{sim}} = \mathrm{curl}\, \mathbf{A}$

$$\int_D \mathrm{curl}\, \nu(\|\mathrm{curl}\, \mathbf{A}\|) \mathrm{curl}\, \mathbf{A} \cdot \mathrm{curl}\, \mathbf{v}\; \mathrm{d}V = \int_D \mathbf{J} \cdot \mathbf{v}\, \mathrm{d}V \;\; \forall \mathbf{v} \in \mathcal{V}$$

$$\nu(s) = \begin{cases} \begin{cases} \frac{1}{s}\left(\hat{f}_{\mathrm{HB}}(s) + \sum_{m=1}^M \sqrt{\lambda_m} \mathbf{y}_m b_m(s)\right) & s > 0 \\ \alpha & s = 0 \end{cases} & \text{in } D_{\mathrm{iron}} \\ 1/\mu_0 & \text{in } D_{\mathrm{air}}. \end{cases}$$

5 Results

Starting with a set of B(H)-data measurements of $K = 26$ specimen in $L = 28$ points, the parametrized H(B) curve is derived. To validate the parameter determination, a ground truth parameter $\mathbf{y}_0 \in [\mathbf{y}^{\min}, \mathbf{y}^{\max}]$ is selected and an artificial dataset $\widehat{\mathbf{B}}_y^{\mathrm{meas}}(p, j)$ of an H-shaped accelerator magnet with $j \in [20{,}450]$ Amp. is generated by solving the magnetostatic problem. Subsequently, the parametrized H(B) curve $f_{\mathrm{HB}}(\mathrm{argmin}\, \mathbf{y}, s)$ is computed. Its relative error compared to $f_{\mathrm{HB}}(\mathbf{y}_0, s)$ is shown in Figure 1. The error in the reconstructed air gap field remains below 10^{-4} T for $j \in [20{,}600]$ Amp. The parametrized H(B) curve and the parameter determination by optimization are suitable to retrieve the H(B) dependence in a built magnet and to adjust the field simulation.

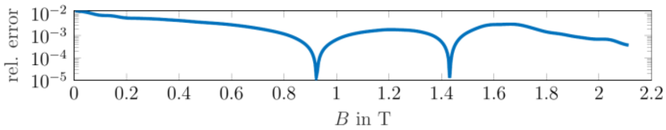

Fig. 1: Relative error between $f_{\mathrm{HB}}(\mathrm{argmin}\, \mathbf{y}, s)$ and $f_{\mathrm{HB}}(\mathbf{y}_0, s)$ in Ampere.

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